II.

OBSERVATIONS OF THE TRANSIT OF VENUS, DECEMBER 5 AND 6, 1882, MADE AT THE HARVARD COLLEGE OBSERVATORY.

By Edward C. Pickering.

Presented December 13th. 1882.

The chances of cloudy weather at this Observatory early in December are large, and Cambridge was not selected by the United States Commission on the Transit of Venus as a station for observations of the phenomenon. It therefore seemed injudicious to make any extensive preparations for the occasion. The available telescopes at the Observatory, however, were employed in observing the contacts. Photometric and spectroscopic observations were also obtained with the East Equatorial, and measurements of the diameter of Venus were made with the telescope of Mr. Chandler, mounted in the West Dome, and also with the East Equatorial.

The morning of the transit was so cloudy that there seemed little prospect of observing the contacts; but the sun gradually became visible, and the clouds were thin enough at the time of ingress to allow observation of both the first and the second contacts. The first part of the afternoon was nearly clear, and the third contact was well seen. A few minutes later the sun entered a mass of thin clouds, but was still sufficiently well seen for observation of the last contact.

Arrangements had been made before the day of the transit with the Western Union Telegraph Company for the distribution of the time signals of this Observatory among those who might desire to obtain them on December 6. The clock at which these signals originate was carefully compared with the standard sidereal clock of the Observatory at frequent intervals, and also with the signals furnished by the United States Naval Observatory at Washington, which were received here by telegraph. To determine the error of the sidereal clock, observations were made with the meridian circle by Professor W. A. Rogers, the results of which are given below. Since the transit, in order to remove any doubts with regard to the error of the sidereal clock as determined by a large fixed instrument, Professor Rogers has

made a special series of observations, in which he used both the meridian circle and the portable transit instrument on each of eight evenings, determining the clock error independently with each instrument. The result confirms the correctness of the form of level employed with the meridian circle, and shows that the instrument furnishes trustworthy results for the absolute as well as the relative clock error. The mean correction to be applied to the error found by the meridian circle in order to reduce it to that found by the portable transit instrument, according to these observations, is $+0^s.08$; the eight separate results are $+0^s.20$, $+0^s.06$, $+0^s.09$, $+0^s.07$, $+0^s.06$, $+0^s.11$, $+0^s.09$, $+0^s.05$, the first having a weight of one third. As the magnifying powers and the reticules used with the two instruments differ materially, the amount of the correction is not surprising.

The results for the error of the sidereal clock obtained from the observations with the meridian circle near the time of the transit are exhibited in Table I. The first two columns contain the date of the observations in mean solar days and tenths, and the sidereal time, to hundredths of an hour, for which the error was determined. The third column gives the number of stars on which each result for clock error depends. The next two columns give the amount by which the clock was slow at the time of each set of observations, and the corresponding error for noon of December 6, corrected by means of the hourly rate $+0^s.023$. The last two columns contain the values of the instrumental constants n and b (angle at pole, and inclination of axis).

TABLE I.— OBSERVED CLOCK ERRORS.

Date. 1882.	Sid. Time.	No. of Stars.	Clock Slow.		n	b
			Observed.	Red. to Dec. 6.0		
Dec. 4.2	$21^h.92$	5	$+2^m\ 21^s.25$	$+2^m\ 22^s.24$	$-1^s.10$	$+0^s.77$
Dec. 4.8	12.63	4	+2 21.69	+2 22.46	−1.03	+0.77
Dec. 5.4	3.07	4	+2 22.05	+2 22.37	−1.05	+0.77
Dec. 6.2	22.05	7	+2 22.36	+2 22.24	−1.05	+0.77

The mean result for noon of December 6 is $+2^m\ 22^s.33$. Reducing this to the result to be expected from the portable transit instrument, by adding $+0^s.08$ as above, we have $+2^m\ 22^s.41$, with an hourly increase of $+0^s.023$.

On December 5, 6, and 7, at noon, the Washington signals were received at Cambridge, and compared by chronograph with our sidereal clock. The result, after allowing for the difference in longitude, was that the Washington clock was fast $0^s.6$, $0^s.4$, and $0^s.3$ on the three days respectively. The signals were promised for December 4 also, but were not received, as the lines were occupied in transmitting political news. A good example is thus afforded of the importance of depending on the local observatories for supplying the public with time.

The clock distributing the mean-time signals from the Harvard College Observatory is kept as nearly as may be $15^s.5$ fast. The time is therefore that of the meridian passing through the State House in Boston, and $4^h\ 44^m\ 15^s.5$ west of Greenwich. On the day of the transit the deviation of the Washington signals was noted, and, to avoid the confusion arising from two systems, our signals were brought to an approximate agreement with them, rather than with our own determination of the local time. Frequent comparisons were made with our sidereal clock, and showed that at December 5.8 our signals were $0^s.6$ fast; at December 6.0, $0^s.5$ fast; at December 6.3, $0^s.5$ fast; and at December 6.8, $0^s.2$ fast. Allowing for the difference of longitude, these signals therefore did not differ more than a tenth of a second from the Washington signals, but to reduce them to the true time both should be regarded as about $0^s.5$ fast. In other words, in reducing to Greenwich mean time, the longitude for the Washington and Boston signals should be taken as $5^h\ 8^m\ 11^s.7$ and $4^h\ 44^m\ 15^s.0$ respectively. Since the observed times of contact are known to be liable to variations of several seconds, these corrections in any case are small, and may be neglected without serious error, especially as it is useless to give the resulting times of contact more closely than to single seconds.

Contacts.

A statement of the results of the contact observations is given below, in Table II. The upper part of the Table contains in successive columns the names of the observers and recorders, the apertures and focal lengths of the telescopes in centimeters, their magnifying powers in diameters, and the corrections required at ingress and at egress to reduce the observed times to Cambridge mean or sidereal time according to the timepiece employed. These corrections are given in accordance with the assumption that the signals furnished by the mean-time clock give the time of a meridian $4^h\ 44^m\ 15^s.5$ west of

Greenwich. The second part of the Table gives the observed times of the four contacts, without any corrections. The third part contains the concluded Greenwich mean times of the contacts noted by each

TABLE II. — CONTACTS.

Observer.	Recorder.	Aperture in cm.	Focal Length in cm.	Power in Diameters.	Corrections of Time-piece.
E. C. Pickering	A. W. Cutler	14.5	682.5	206	+9s.0, +10s.3
Arthur Searle	W. A. Rogers	13.2	230	220	—76s.2
O. C. Wendell	A. W. Cutler	10.2	141.4	40, 90	+9s.0, +10s.3
J. R. Edmands	R. G. Saunders	10.2	140	150	+82s.3, +82s.3
S. C. Chandler, Jr.	W. V. Brown	15.2	244	180	—26s.8, —24s.7
W. H. Pickering	R. G. Saunders	6.4, 10.2	71, 84	20, 110	+82s.3, +82s.3

Observer.	Observed Times of Contacts.			
	I.	II.	III.	IV.
E. C. P.	21h 19m 43s.4	21h 39m 51s.3	3h 3m 3s.8	3h 23m 10s.0
A. S.	. . . . .	. . . . .	20 6 23.4	20 26 28
O. C. W.	21 20 8.2	. . . . .	3 3 8.2	3 23 14.2
J. R. E.	. . . . .	14 40 1	20 3 40	20 23 40
S. C. C.	21 20 22	21 40 30	3 3 30	3 24 10
W. H. P.	. . . . .	14 40 9	20 3 33	20 24 0

Observer.	Greenwich Mean Time of Contacts.				Difference from Mean.			
	I.	II.	III.	IV.	I.	II.	III.	IV.
E. C. P.	2h 4m 23s	2h 24m 31s	7h 47m 45s	8h 7m 51s	—9s	—12s	+5s	—1s
A. S.	. . .	. . .	7 47 41	8 7 42	. . .	. . .	+1	—10
O. C. W.	2 4 48	. . .	7 47 49	8 7 55	+16	. . .	+9	+3
J. R. E.	. . .	2 24 50	7 47 36	8 7 33	. . .	+7	—4	—19
S. C. C.	2 4 26	2 24 34	7 47 36	8 8 16	—6	—9	—4	+24
W. H. P.	. . .	2 24 58	7 47 30	8 7 54	. . .	+15	—10	+2
Mean	2 4 32	2 24 43	7 47 40	8 7 52				

observer, obtained by adding 4^h 44^m 31^s to each of the Cambridge mean times. The mean result for each contact is given in the last line of the Table. At the right are given the differences of each observer's result from the mean of all.

The following notes contain, under the name of each observer, the details of his work.

E. C. Pickering.

The instrument employed was the East Equatorial. Its full aperture is 15 inches, which on this occasion was reduced to about 6 inches by a cap over the object-glass. An audible signal was given to the recorder at the time of each phenomenon noted. The recorder took the time of each signal from the chronometer, and recorded it, with any subsequent remarks by the observer. The wedge of shade glass placed between the eyepiece and the eye was of a greenish tint. In observing the first contact, the last time recorded before the appearance of the notch was 9^h 19^m $29^s.6$ by the chronometer. Venus was first seen at 9^h 19^m $44^s.2$ by the chronometer. The edge of the sun was wavy, rendering it difficult to decide whether an indentation was real. At 9^h 19^m $49^s.0$ the interval between the cusps was estimated at $9''$; two parallel lines $6''$ apart served as the unit of measure. From a reduction of this observation the time of first contact appears to be 9^h 19^m $42^s.6$; the mean of this and of the time directly observed is here assumed to be the time of the first contact, which is therefore 9^h 19^m $43^s.4$ by the chronometer. A scale in the eyepiece would allow the observer of phenomena like these to make estimates of the interval of the cusps without removing his eye from the telescope, and would accordingly afford him many of the advantages of a double-image micrometer without its disadvantages.

The images at the second contact were unusually well defined, and the contact was recorded as occurring at 9^h 39^m $51^s.3$. Eight seconds later it was clearly past.

The third contact was recorded as occurring at 3^h 3^m $3^s.8$. At 3^h 3^m $21^s.4$ the interval between the cusps was estimated as double that between the lines in the field, and consequently as $12''$. A reduction of this observation would make the time of contact 3^h 3^m $0^s.0$.

The fourth contact was recorded as occurring at 3^h 23^m $10^s.0$. At 3^h 23^m $3^s.4$ it had not occurred, at 3^h 23^m $19^s.4$ it was certainly past.

The chronometer used by the recorder, who also recorded for Mr.

Wendell, was Bliss & Creighton 1182; it is regulated to mean time, and has been in frequent use at the Observatory.

Arthur Searle.

The instrument was the five-inch telescope formerly mounted in the West Dome. As it was not provided with any stand, and as economy was an object kept strictly in view during the preparations for the transit, the best plan for using this telescope seemed to be to lay it horizontally upon a rough frame, at a height of three feet from the ground, before the south entrance of the Observatory. A plane mirror of unsilvered glass, formerly used in photographing the sun, was placed upon the block of stone at the east side of the steps of the entrance. This mirror was attached to the frame originally prepared for it, which is provided with screws for moving it approximately in altitude and in azimuth. The dimensions of the mirror are $7\frac{1}{2}$ by 6 inches, so that the sunlight reflected from it at moderate hour angles was thrown upon the whole surface of the object-glass before it. The two surfaces of the mirror are inclined to each other, so that only one image of the sun is seen. In order to keep this image of the sun in the field, the services of an assistant were necessary. Unfortunately, the assistant who had accustomed himself before the transit to the management of the mirror, considering the morning too cloudy for any observation, did not arrive at the Observatory in season to take part in the observations at ingress. No other assistant having the necessary skill was available, and an attempt made to use the telescope at ingress was therefore unsuccessful.

At egress, the mirror was very successfully managed so as to keep the required part of the sun's limb in view, and Professor W. A. Rogers kindly undertook to record the times at which the observer gave his signals. The observation of the third contact was accordingly a satisfactory one. Nine seconds before the time recorded as that of the contact, the sun's limb became noticeably darkened at the place of egress, but the shade was lighter than the tint of the planet itself. The shade gradually darkened as the planet advanced, and at the time recorded as that of contact a darkness equal to that of the planet's disk had reached the limb of the sun. This phenomenon could not be distinguished from that of geometrical contact. If the limb of the sun had been steadier, it is possible that such a distinction might have been made. The image, in fact, was by no means bad, but there was sufficient undulation to make a very exact observation of geometrical contact impossible. Thirteen seconds after the recorded time it had

become evident that the cusps were separated by a part of the planet's limb, and that geometrical contact was past.

These observations were made through a dark red shade-glass between the eyepiece and the eyestop. The limb of the sun was distinctly seen, and was free from glare. As the sun entered the thin clouds mentioned in the introductory remarks above, the red glass was replaced by a blue one, which admitted much more light. The part of the planet exterior to the limb of the sun was then certainly, though indistinctly, seen. Its outline seemed to be part of a smaller circle than that bounding the portion of the disk interior to the sun's limb. The increasing cloudiness soon put an end to this appearance, the Greenwich mean time of which, derived from the record, is $7^h\ 57^m\ 31^s$.

The fourth contact was observed with some difficulty, owing to the clouds and to the necessity of an occasional movement of the mirror to keep the image in the field. The time given is that of a signal accompanied by the remark "Notch doubtful"; the notch was not afterwards seen.

During the transit, the disk of the planet was uniformly dark, except that at times it seemed to be crossed by faint streaks of light, very likely due to slight defects in the shade-glass or other parts of the optical apparatus employed.

The eyepiece used was positive, No. 5 of the set of eyepieces belonging to the large filar micrometer of the East Equatorial, and, with that instrument, having a nominal magnifying power of 688. Its power, with the telescope used during the transit, has been determined by two methods, and the approximate mean result 220 is given in the Table.

The chronometer employed was Bond 236, regulated to sidereal time; it has been in constant use at the Observatory for many years.

O. C. Wendell.

The instrument employed was the finder of the East Equatorial. The object-glass was silvered to reduce the light, and an additional reduction was effected by a shade-glass. Between the first and second contacts the silvering was partially removed, owing to an apprehension that the clouds would grow thicker; but as the sky actually became clearer, the second contact could not be observed. Before the third contact, the film of silver was entirely removed, and the object-glass was smoked by Mr. Clacey's method, which sufficed, with the aid of one shade-glass, to reduce the light.

The first contact was well observed, but the recorder did not notice the signal, and the time is derived by estimate. According to the observer, the signal was given half-way between the last two signals of Professor Pickering, whose own estimate, however, placed it six seconds later. The mean of these estimates was adopted.

The third and fourth contacts were well seen and recorded. The time given for the third contact is that when the diminishing thread of light at the place of egress definitely broke. Eighteen seconds earlier contact had certainly not yet occurred.

Eight seconds previous to the time of fourth contact it was evident that the egress had not been completed.

J. R. Edmands.

The telescope was one borrowed from Dr. E. T. Caswell, of Providence, R. I., and originally owned by Dr. Alexis Caswell, of Brown University. It was attached to a portable equatorial mounting (without clock) belonging to the Observatory. The observations at ingress were made on the east balcony of the dome, and at egress on the west balcony. The eyepiece was negative, and the light was reduced by one shade-glass placed near the focus of this eyepiece at second contact, and by three shade-glasses, one on each side of the focus and one next the eye, at the third and fourth contacts.

The chronometer used in recording these observations, as well as for those of Mr. W. H. Pickering, was Frodsham 3451, regulated to sidereal time. This is an excellent instrument, of much value in the work of the Observatory Time Service.

At first contact the rapid changes in the opacity of the clouds prevented observation of the phenomenon, as the observer had no wedge of shade-glass, and could not readily control the brightness of the field. At second contact, the recorder found it impracticable to note the times from the signals of the two observers, and the assumed times are derived by estimate. The original estimate of the time of the signal "Past" was derived from an inspection of the chronometer made immediately after the observer learned that no record had been secured. But he was satisfied, on consideration, that this first estimate allowed too little for the interval between the signal and his inspection of the chronometer; besides which, his signal "Past" must have been given an appreciable time after the contact itself. His last signal, "Not yet," preceded the signal "Past" by about ten seconds. On these accounts, he estimated the observed time of contact, which has been entered in the Table, as five seconds earlier than the original

estimate of the time of the signal "Past." This decision was made before correcting the result for error of chronometer, and before comparing it with any other observation.

At the third and fourth contacts, the recorder counted the seconds from the chronometer, and the observers recorded the times of their observations. At the third contact, the seeing was good, and the following note was made: "No black drop seen. Purposely used faint image." The clouds impeded any similar observation at the second contact. The telescope was somewhat disturbed by wind during the observations at egress, which prevented the observation of additional phenomena.

S. C. Chandler, Jr.

The first contact was looked for at a part of the limb estimated to be 3° to 5° to the right of the apparent vertex. The notch was suddenly noticed still nearer the apparent vertex at the time given as that of contact. Nineteen seconds later the notch had increased, confirming the first observation with regard to the place of ingress.

The time given as that of second contact is that of geometrical contact in the opinion of the observer. Seven seconds before the recorded time the contact had not occurred. Four seconds after the recorded time it was still uncertain whether the contact was past. Nine seconds after the recorded time the contact was certainly past. No "black drop" was seen.

Thirty-five seconds before the recorded time of third contact a shade appeared on the sun's limb, very much fainter than the disk of Venus. This shade increased in darkness, but did not seem to confuse the determination of the time of geometrical contact, which is that recorded for this phase of the transit. The recorded time must be as early as that of geometrical contact, which might possibly, however, have been thought to occur ten seconds later than the recorded time. Twenty-two seconds after the recorded time, contact was certainly past by several seconds.

The fourth contact was very satisfactorily observed. Eight seconds before the recorded time the notch was still certainly visible; at the recorded time it was certainly gone.

The telescope was one belonging to the observer, and lately placed on the equatorial mounting in the West Dome. No shade-glasses were used. The necessary reduction of the light was effected by previously smoking the front surface of the crown and the back surface of the flint lens of the object-glass. This was done by Mr. John Clacey,

the maker of the telescope, and the result proved very satisfactory. A negative eyepiece was used in observing the contacts.

The timepiece used was the pocket-chronometer Patek, Phillipe, & Cie. 34,807.

W. H. Pickering.

The instrument selected for the observations was the Bowditch Comet-seeker; but as dew upon its reflecting prism prevented observations with it at ingress, the Quincy Comet-seeker, a smaller instrument, was employed in observing the second contact. No complete record of these observations was secured, owing to the circumstances explained in the notes relating to Mr. Edmands's observations. The estimate of the time of contact is based upon the circumstance that the observer's signal "Past" was given one second later than the corresponding signal by Mr. Edmands, according to the judgment of both observers. But on consideration, previous to any comparison with other observations, it appeared likely that this signal "Past" was given a little too early. The observer, therefore, assigned for his observation of contact a time three seconds later than the original estimate of Mr. Edmands for the time of his own signal.

The Bowditch Comet-seeker was used at egress.

Observers at other Stations.

The following observations of the transit have been communicated to me for publication, and are here inserted.

1. Station, the establishment of Messrs. Alvan Clark & Sons in Cambridgeport, Massachusetts. Approximate latitude, $+42° 21' 16''$; approximate longitude, west of Greenwich, $4^h 44^m 26^s.7$. The times are given according to the clock signals of this Observatory.

Observer, Alvan G. Clark. Second contact, $21^h 40^m 3^s$; third, $3^h 2^m 30^s$ (the observer has no doubt that the minute should be 3 instead of 2); fourth, $3^h 23^m 54^s.5$.

Observer, C. A. R. Lundin. Third contact, $3^h 3^m 13^s$; fourth, $3^h 23^m 34^s$.

Reducing these observations to Greenwich mean time by the addition of $4^h 44^m 15^s.5$, we have, for Mr. Clark, $2^h 24^m 18^s$, $7^h 47^m 46^s$, $8^h 8^m 10^s$; and for Mr. Lundin, $7^h 47^m 28^s$, $8^h 7^m 50^s$.

2. Station, near St. Paul's Church, New York. Approximate latitude, $+40° 46'.0$; approximate longitude, west of Greenwich, $4^h 56^m 0^s$. Observer, Rev. G. M. Searle. Telescope by Dollond; aperture, 2.65 inches; focal length, 44 inches; magnifying power, 60.

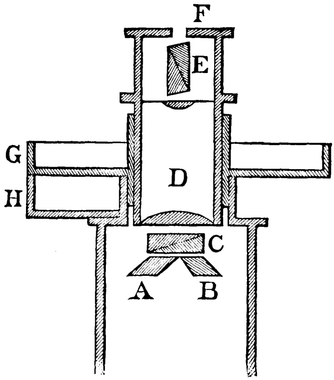
Timepiece, a good watch, the errors of which were determined by sextant observations at $22^h\ 16^m$, 22^h 34 , and $1^h\ 11^m$, which gave the respective corrections $+9^s$, $+8^s$, $+3^s$. No "black drop" was seen at either internal contact. The first contact was lost; the rest were observed as follows: $21^h\ 28^m\ 2^s$, $2^h\ 51^m\ 49^s$, $3^h\ 11^m\ 50^s$. The corrected mean times are $21^h\ 28^m\ 10^s$, $2^h\ 51^m\ 52^s$, $3^h\ 11^m\ 53^s$; and the corresponding Greenwich mean times are $2^h\ 24^m\ 10^s$, $7^h\ 47^m\ 52^s$, $8^h\ 7^m\ 53^s$. The sun's limb was remarkably steady at egress, but somewhat disturbed at ingress.

3. Station, terrace at No. 55 Habana Street, Havana, Cuba. Approximate latitude, $+23^\circ\ 9'\ 21''$; approximate longitude, west of Greenwich, $5^h\ 29^m\ 26^s$. Observer, Professor Charles Hasselbrink (U. S. Signal Service observer in Havana). Telescope by Negretti and Zambra; aperture, 2.5 inches; focal length, 39 inches; magnifying power, 80. Chronometer Negus 582, slow 3^s, by comparisons furnished by the observatory of Don José Maria Garcia de Haro, semi-official observer for the Spanish Navy and the mercantile marine. The observer recorded for himself; the telescope was shaken by wind in the afternoon. The external contacts are considered doubtful, but the internal contacts were well observed. Observed times of contacts, $20^h\ 33^m\ 57^s$, $20^h\ 54^m\ 30^s$, $2^h\ 19^m\ 0^s$, $2^h\ 36^m\ 47^s$; corrected mean times, $20^h\ 34^m\ 0^s$, $20^h\ 54^m\ 33^s$, $2^h\ 19^m\ 3^s$, $2^h\ 36^m\ 50^s$; resulting Greenwich mean times, $2^h\ 3^m\ 26^s$, $2^h\ 23^m\ 59^s$, $7^h\ 48^m\ 29^s$, $8^h\ 6^m\ 16^s$.

Just before internal contact at ingress, the observer saw a fine line of light round the disk of Venus, beyond the limb of the sun. During the transit, a delicate aureola of very white light was noticed around the planet, suggesting the illumination of its atmosphere. Patches of a dark grayish tint were noticed at times upon the deep black disk of Venus.

Photometric Observations.

A photometer was constructed for comparing the brightness of the disk of Venus during transit with that of the sky in immediate proximity to the sun's limb. In the accompanying figure, *A* and *B* are two glass prisms, the first having parallel sides, the other with sides inclined at a small angle. *C* is a double-image prism, *D* a positive eyepiece, *E* a Nicol, and *F* an eyestop. A graduated circle, *G*, and an index, *H*, serve to measure the angle through which the eyepiece and Nicol are turned. The whole is inserted, like an eyepiece, in the tailpiece of the 15-inch telescope of the Observatory. The light from the object-glass, striking upon the prism *A*, is not deviated, but is divided by the prism



C into two pencils, one of which passes without deviation through the eyepiece and the hole in the eyestop to the eye. The other pencil is thrown to one side by *C*, and is cut off by the eyestop. The light passing through *B* is deviated about 6° by the difference in inclination of its two inclined sides. This light is also divided into two pencils by *C*, one retaining the deviation imparted by *B*, and being cut off by the eyestop. The other is deviated by *C*, but in such a manner as to counteract the inclination imparted to it by *B*. It therefore passes centrally through the hole in the eyestop to the eye of the observer. The latter accordingly receives two pencils of light formed by the same object-glass, one receiving the light from *A*, the other that from *B*. These two pencils are polarized by *C* in planes at right angles, and their relative brightness may accordingly be varied at will by turning the Nicol *E*. The instrument in principle closely resembles the meridian photometer for some years in use at this Observatory. The same device is employed to secure two equal pencils polarized in perpendicular planes, but in that instrument two equal object-glasses are employed, instead of two images of the same objective.

The eyepiece is focused on the front surface of the prisms *A*, *B*, so that their adjacent edges appear as a line dividing the field into two equal parts. By turning the Nicol the brightness of either part of the field may be reduced indefinitely, so that the brighter may always be brought to equality with the fainter. Placing the whole instrument at the principal focus of the telescope, we see side by side in the two halves of the field images of objects really about 16′ apart.

The observations were made by placing the edge of the prism parallel to the sun's limb at the point nearest Venus, and bringing Venus into one half of the field. A portion of the sun's disk near its centre will be seen in the other half of this field, and may be compared directly with Venus by turning the Nicol. Settings were made in the four positions of the Nicol in which the images appeared equal, and the positions read to tenths of a degree. The observation was then repeated, moving the telescope so that the portion of the sky close to the sun's limb should be measured in the same manner. Eight settings taken

in this way constitute a set, and give the relative light of the sky and Venus. To eliminate any difference in the prisms the photometer was rotated 180° after each set, but no perceptible difference is indicated in this way. To reduce the observations, the first reading was subtracted from the second, and the third from the fourth. Calling the sum of these differences A, the relative light, $L = \tan^2 \frac{1}{4} A$. It will also be convenient to use a method of expressing the light in stellar magnitudes, according to the method already used in this Observatory for comparing nebulæ and portions of the moon. When surfaces are thus compared, portions of equal area are selected and reduced to stellar magnitudes by the formula of Pogson. We shall then have the difference in magnitude, $M = 2.5 \log L$. In Table III. the successive columns give a current number, the Cambridge mean time, the difference in light of equal areas of Venus and the sun expressed in stellar magnitudes, the corresponding quantities for the sky near the edge of the sun, and these same ratios expressed in percentages, that is, assuming the light of the centre of the sun equal to one hundred. The last column gives the initial of the observer.

TABLE III. — PHOTOMETRIC OBSERVATIONS.

No.	Cambridge Mean Time.	Difference in Magnitude.		Percentages.		Observer.
		Venus.	Sky.	Venus.	Sky.	
1	$1^h\ 7^m.6$	4.30	2.54	1.9	9.6	W.
2	1 18.4	4.55	2.94	1.5	6.7	W.
3	1 24.8	4.05	2.79	2.4	7.7	W.
4	1 29.4	4.46	2.40	1.6	11.0	W.
5	2 48.3	5.58	3.32	0.6	4.7	P.
6	2 51.6	4.44	2.74	1.7	8.0	P.
7	2 54.4	4.37	3.06	1.8	6.0	P.
8	Mean W.	4.34	2.67	1.8	8.8	W.
9	Mean P.	4.80	3.04	1.4	6.2	P.
10	Mean W., P.	4.57	2.85	1.6	7.5	Both.

The result for the light of the sky in the first line of the table depends upon eight settings.

These observations show a well-defined increase in light of the sky near the edge of the sun as compared with that received from Venus. This effect also seemed to me to be very perceptible without the photometer. To confirm it, I asked Mr. Wendell which looked to him the brighter. He satisfied himself that Venus certainly appeared darker than the sky. A slight difference was to be expected, since there are instances on record of the visibility of Venus before first

contact. In the interval between the exterior and interior contacts, the edge of the planet has sometimes been traced beyond the limb of the sun. The effect was noticed by Mr. Searle shortly after the third contact, as above stated. According to his recollection of the appearance of the field, the difference in darkness beween the planet's disk and the sky was obvious, and might have been expected to make the planet more distinctly visible outside of the sun's limb than was actually the case.

No appreciable light could be received from Venus itself, unless that planet is incandescent or phosphorescent, an extremely improbable hypothesis. Doubtless the greater portion of the light, like that of the sky near the sun, is due to reflection of the light of the sun from the particles of the earth's atmosphere. During a total solar eclipse the interposition of the moon suffices to cut off nearly all the light near the sun, except the small portion due to the solar corona. It is therefore obvious that the light of the sky near the sun originates at no great distance from the earth, and is doubtless caused by reflection in the terrestrial atmosphere. In a communication to this Academy nine years ago (*Proceedings*, IX. 1), I showed that many of the phenomena of atmospheric illumination and polarization could be explained by specular reflection from the particles of the air, whose index of refraction differs very slightly from unity. In this case, if the sun was reduced to a point, the light of the sky at small distances would vary inversely as the fourth power of the distance. In any case, a glance towards the sun is sufficient to show that the light increases very rapidly as we approach the sun's limb. We should expect that the light of the portion of the atmosphere between us and the sun would be much greater than that outside of the sun's disk. Most of the light would be received from the portion of the sun at a very small angular distance. A point between us and the sun would be illuminated in all directions, that is, through the entire 360°. A point outside the sun's disk could at most receive light only from 180°. Moreover, the edge of the sun is much fainter than its centre, which would still farther reduce the light. We should then expect that the light received from Venus would be greater than that of the sky near the sun's limb, the opposite result from that indicated by the observations. This effect would be modified by the solar atmosphere, which would increase the light outside of the sun. The observations of Professor Langley, however, during the eclipse of 1878, seem to prove that the light of the corona is entirely insufficient to produce this effect. The difficulty of photographing the corona confirms this view, but its spectrum in-

dicates that it is composed of light of a wave-length to which the photographic plate is not very sensitive.

An important source of error arises in almost all these measures from the diffuse reflection from dust or scratches on the object-glass. The effect of this would be similar to an increased haziness of the sky, and would tend to increase the apparent light received both from Venus and the sky. In our measures this effect was reduced to a minimum, as the object-glass had been cleaned shortly before the transit, and the diffuse reflection was therefore very slight. A remedy for this difficulty would be found by removing the object-glass and substituting for it a minute hole. When the sky is hazy we should expect an increased relative brightness near the edge of the sun. This may account for the larger readings obtained by Mr. Wendell, as the sky was somewhat clearer during my observations than during his. As the portion of the sky observed was only about 1′ distant from Venus, irregular clouds could not produce the observed difference in light. In fact, the persistence of the phenomena under varying conditions seems to leave little doubt that the disk of Venus was really much darker than the sky near it.

The solution of this problem would be greatly aided by researches of the kind described below, a portion of which will probably be undertaken at this Observatory. Measurement of the relative light of different portions of the sun's disk and of the sky at various distances from it. Measures of the sky at various distances from the moon, thus eliminating any effect corresponding to that of a solar atmosphere. Measures of the light of the disk of the moon during the progress of a partial eclipse of the sun.

Spectroscopic Observations.

The spectrum of the light received from Venus was observed with a star spectroscope constructed by Hilger. The dispersion was such that about one third of the spectrum was visible in the field of view at a time. Two prisms of dense flint glass were employed. This spectroscope was attached to the large Equatorial, and was focused on the limb of the sun. When Venus was brought upon the slit it appeared as a broad band traversing the spectrum lengthwise, which could be compared directly with the solar spectrum on each side of it. The slit was then brought tangential to the limb of Venus, so as to receive the light grazing its surface. The breadth of the dark band was thus reduced and flickered owing to the slight unsteadiness of the

atmosphere. No difference in the spectrum was detected either by myself or by Mr. W. H. Pickering after careful examination.

This negative result should not be regarded as throwing doubt on the positive results attained by so skilful an observer as Professor Young, who is said to have detected the presence of aqueous vapor in Venus. I have not yet seen the details of his observation, but his facilities for making this observation were much greater than mine, and he probably used a much higher dispersion. I satisfied myself that there were no very marked absorption bands, and doubtless the phenomenon is one which requires more careful preparation than we were permitted to make without interfering with the other portions of our programme to which, in preparing our plans, we had attached more importance.

Diameter of Venus.

The measurements of the diameter of Venus, mentioned in the first paragraph of this communication, were made by Professor William A. Rogers and by Mr. S. C. Chandler, Jr. The subjoined reports from these gentlemen furnish the account of the work undertaken. In these reports, Mr. Chandler's telescope, mounted in the West Dome, has been called the West Equatorial.

Report by William A. Rogers.

The following method for the determination of the diameter of a planet was first employed by the writer in 1877, having been used in the determination of the diameter of Mars.

Let: —

x_0 = a line ruled upon glass and set in the direction of diurnal motion.

x_1 = a line ruled at a given angle, i, with respect to x_0, and reckoned from east to west.

x_2 = a line ruled at the angle $(180° - i)$ with respect to x_0.

y = a line ruled at right angles to x_0 and bisecting the angle formed between x_1 and x_2.

τ_1 = the observed time of transit of the preceding limb of the planet over x_1.

τ_2 = the time of transit of the following limb over x_1.

τ_3, τ_4 = the corresponding times over x_2.

D = the diameter of the planet.

Then:

$$D = 15 \cos \delta \, (\tau_2 - \tau_1) \sin i = 15 \cos \delta \, (\tau_4 - \tau_3) \sin i$$

For any variation whatever of the angle i we have:

$$\Delta i = \frac{D}{15 \cos \delta\, (\tau_2 - \tau_1) \cos i}$$

and hence, from transits over the line x_1,

$$D = 15 \cos \delta\, (\tau_2 - \tau_1) \sin\, (i + \Delta i)$$

$$= 15 \cos \delta\, (\tau_2 - \tau_1) \sin \left[i + \frac{D}{15 \cos \delta\, (\tau_2 - \tau_1) \cos i} \right]$$

and from the transits over the line x_2,

$$D = 15 \cos \delta\, (\tau_4 - \tau_3) \sin \left[i - \frac{D}{15 \cos \delta\, (\tau_4 - \tau_3) \cos i} \right]$$

If therefore the times of transit of each limb are taken over the lines x_1 and x_2, any error in D due to an erroneously assumed position angle will be eliminated.

It must be noted, however, that any error in Δi arising from an unknown error in the angles between x_0, x_1, and x_2 will be only partially eliminated. Designating by i and i' the angles which x_1 and x_2 make with x_0, and their variations on account of errors of graduation by Δi and $\Delta i'$ respectively, we have, from transits over x_1,

$$D = 15 \cos \delta\, (\tau_2 - \tau_1)\, [\sin i + \cos i\, \Delta i]$$

and from the transits over x_2,

$$D = 15 \cos \delta\, (\tau_4 - \tau_3)\, [\sin i' + \cos i'\, \Delta i']$$

or, since $i' = 180^\circ - i$ nearly,

$$D = 15 \cos \delta\, (\tau_4 - \tau_3)\, [\sin i - \cos i\, \Delta i']$$

whence

$$D = \tfrac{15}{2} \cos \delta\, \big[[(\tau_2 - \tau_1) + (\tau_4 - \tau_3)] \sin i + \cos i\, [(\tau_2 - \tau_1)\, \Delta i - (\tau_4 - \tau_3)\, \Delta i'] \big]$$

The only case, therefore, in which the elimination will take place is that in which

$$(\tau_2 - \tau_1)\, \Delta i = (\tau_4 - \tau_3)\, \Delta i'$$

But since, on Dec. 6, the time required for Venus to make a complete transit over a line having $i = 20^\circ$ was only 24^s, the effect of any small error in the graduation will be practically insensible.

For the equatorial diameter we have:

$$D = 15 \cos \delta\, (\tau_2 - \tau_1) \sin\, (90^\circ + \Delta i)$$

Unless Δi, therefore, is very large, we shall have:

$$D = 15 \cos \delta\, (\tau_2 - \tau_1)$$

Assuming the same constant of differential refraction for Venus north and for Venus south, any error in the observed value of D

due to the differential refraction R_{rf} will be eliminated if we combine the observations over x_1 and x_2 with corresponding observations over these lines extended below the line x_0. Designating the times of transit for Venus north of x_0 by τ'_1, τ'_2, τ'_3, and τ'_4, we shall have:

For Venus South.

$$D = \tfrac{15}{2} \cos\delta \left[(\tau_2 - \tau_1) \sin\left(i + \frac{D}{15 \cos\delta\, (\tau_2 - \tau_1) \cos i}\right) + (\tau_4 - \tau_3) \sin\left(i - \frac{D}{15 \cos\delta\, (\tau_4 - \tau_3) \cos i}\right)\right] + R_{rf}$$

For Venus North.

$$D = \tfrac{15}{2} \cos\delta \left[(\tau_2 - \tau_1) \sin\left(i + \frac{D}{15 \cos\delta\, (\tau'_2 - \tau'_1) \cos i}\right) + (\tau_4 - \tau_3) \sin\left(i - \frac{D}{15 \cos\delta\, (\tau'_4 - \tau'_3) \cos i}\right)\right] - R_{rf}$$

Combining these equations, we shall still have, for any case except where Δi is due to an error in the assumed value of i, an equation of the form:

$$D = 15 \cos\delta\, (\tau_2 - \tau_1) \sin i$$

Two ruled plates were prepared for the observation of Dec. 6, one for the East Equatorial and one for the West Equatorial. They consist of one horizontal line, two vertical lines, and a series of lines having the inclinations 10°, 20°, 30°, 40°, 45°, and the inclinations 135°, 140°, 150°, 160°, and 170°, respectively, to the horizontal line. These lines were all extended below the line x_0, giving the angles 225°, 230°, 240°, 250°, 260°, and the angles 315°, 320°, 330°, 340°, and 350°.

In general, a complete series of observations consists of 10 transits over each of the inclined lines, and 20 transits over the vertical lines, both for Venus *south* and for Venus *north* of the horizontal line.

The results for Dec. 6, arranged in the order of the times of observation are as follows.

TABLE IV. — East Equatorial.

Position of Venus with respect to horizontal line.	$i = 30°$		$i = 45°$		$i = 90°$		Remarks.
	D.	No. Obs.	*D.*	No. Obs.	*D.*	No. Obs.	
South.	″ 62.51	18	″ 62.46	18	″ 60.45	32	Seeing fair.
South.	62.27	20	60.85	20	59.62	40	Seeing fair. New zero of position.
North.	59.73	22	60.26	22	60.18	42	Seeing very bad.
South.	59.21	22	58.98	22	58.37	44	Image of Venus boiling. Reject.

TABLE V. — West Equatorial.

Position of Venus.	$i=10°$ D.	$i=10°$ No. Obs.	$i=20°$ D.	$i=20°$ No. Obs.	$i=30°$ D.	$i=30°$ No. Obs.	$i=45°$ D.	$i=45°$ No. Obs.	$i=90°$ D.	$i=90°$ No. Obs.	Remarks.
	″		″		″		″		″		
South.	....	....	61.18	24	60.73	24	61.04	22	60.18	44	Seeing very good.
South.	....	....	61.37	20	60.94	20	61.43	20	60.59	40	Seeing fair. New zero.
North.	....	....	59.52	22	60.53	22	59.28	22	59.35	44	Seeing unsteady.
South.	60.56	22	....	....	....	....	....	....	61.70	44	Seeing fair.
North.	56.51	22	....	....	....	....	....	....	59.62	44	Seeing bad.

Collecting the results for each instrument, but rejecting the last series with the East Equatorial, we have:

TABLE VI.

Instrument.	$i=10°$	$i=20°$	$i=30°$	$i=45°$	$i=90°$
	″	″	″	″	″
East Equatorial	....	....	62.51	62.46	60.45
	....	....	62.27	60.85	59.62
	....	....	59.73	60.26	60.18
Means	....	....	61.50	61.19	60.08
West Equatorial	....	61.18	60.73	61.04	60.18
	....	61.37	60.94	61.43	60.59
	....	59.52	60.53	59.28	59.35
	60.56	....	....	....	61.70
	56.51	....	....	....	59.62
Means	58.54	60.69	60.73	60.58	60.29

Combining the results of the observations over the inclined lines, and assigning the same weight for each value of i, we have:

TABLE VII.

	D for $i=90°$	D for $i=10°....45°$
From the East Equatorial . . .	60.08″	61.34″
From the West Equatorial . . .	60.29	60.28
Means	60.18	60.81
Values of D at the distance unity	15.92″	16.09″

It will be seen, by an examination of Tables IV. and V. that the magnitude of D apparently depends to a certain extent upon the

character of the atmospheric conditions under which the observations were made. Arranging the results according to the character of the seeing, we have:

TABLE VIII. — SEEING FAIR TO GOOD.

Instrument.	$i=10°$	$i=20°$	$i=30°$	$i=45°$	$i=90°$
	"	"	"	"	"
East Equatorial	....	....	62.51	62.46	60.45
	....	....	62.27	60.85	59.62
West Equatorial	....	61.18	60.73	61.04	60.18
	....	61.37	60.94	61.43	60.59
	60.56	....	....	....	61.70
Means	60.56	61.27	61.61	61.45	60.51
		SEEING BAD TO VERY BAD.			
East Equatorial	....	....	59.73	60.26	60.18
	....	....	59.21	59.98	58.37
West Equatorial	....	59.52	60.53	59.28	59.35
	56.51	....	....	....	59.62
Means	56.51	59.52	59.82	59.84	59.38

Combining by weights proportional to the number of observations, we have:

TABLE IX.

	D for $i=90°$	D for $i = 10° \ldots 45°$
	"	"
For seeing fair to good . .	60.51	61.39
For seeing bad to very bad	59.38	59.38

From the observations made under favorable conditions, we have for the distance unity:

D for $i=90°$	D for $i=10° \ldots 45°$
$16''.01$	$16''.24$

There is a general tendency of the observations to indicate a lesser value for the equatorial diameter, but the method of obtaining this quantity by direct transits over a vertical line is not a very reliable one. The apparent difference, therefore, between the diameter determined at different angles of inclination, is probably fictitious rather than real.

In order to determine the difference in the amount of the irradiation

of a dark disk upon a bright ground and of a bright disk upon a darker ground, observations for the diameter were continued for several days succeeding the transit. Since it was only possible to observe both points of tangency of the inclined lines with the disk of the planet on one side of the vertical lines, the elimination of the effect of an error in the position angle of the line x_0 does not here take place. Care was taken, however, to make the setting for the zero of position as exact as possible.

The following results were obtained.

TABLE X.

Date.	Instrument.	$i = 10°$	$i = 20°$	$i = 30°$	$i = 40°$	$i = 45°$	Means.
1882.		″	″	″	″	″	″
Dec. 13–14	E. Equatorial	....	....	16.75	16.45	16.49	16.56
Dec. 14–15	E. Equatorial	....	....	16.66	16.58	16.14	16.46
Dec. 24–25	E. Equatorial	....	....	16.90	17.33	....	17.12
Dec. 26–27	E. Equatorial	....	17.96	17.66	17.47	17.29	17.47
1883.							
Jan. 1–2	West Equatorial	17.73	....	....	....	....	17.73

If these observations can be trusted we may conclude: —

(*a.*) That the difference in the value of the diameter at the distance unity, due to irradiation on Dec. 6, and on the days immediately following, is not far from 0″.4.

(*b.*) That up to a certain point this difference increases with the angular increase in the distance of the planet from the sun. This increase, however, is probably not quite as great as the observations seem to indicate. On Dec. 26 and Jan. 1 the atmospheric conditions were not favorable to good observations.

Report by S. C. Chandler, Jr.

The following determinations of the diameter of Venus during the transit on Dec. 6, 1882, were made by Professor Rogers's plan of transits over inclined lines, with the West Equatorial. The telescope had been prepared for solar observation by the maker, Mr. John Clacey, by smoking the front of the crown and the back of the flint lens of the object-glass; a process which he finds affords a better effect than a silver film, the image being sharper and the effect of contrast with the sky more agreeable. The result in the present instance was completely satisfactory. The obscuration produced by the double smoke film was sufficient to render a shade glass unnecessary with the power used, which was about 180 diameters.

The scheme of observation, and the plate, were the same as used by Professor Rogers. The transits were taken in sets consisting of an equal number of contacts of both limbs with lines ruled at equal and contrary angles with the middle transit lines, thus eliminating the error of the zero of position. The formula of reduction follows simply from equation (2) hereafter given. Thus, if we call Δt_1, Δt_2, the differences of the observed times of transit of opposite limbs, for the angles p and $-p$, respectively, we get

$$D = \tfrac{15}{2} \cos \delta \cos p \, (\Delta t_1 + \Delta t_2).$$

The corrections for proper motion and differential refraction are so far within the uncertainty of observation, in their effect on the concluded diameter, that they have been neglected. Table XI. gives the value of the observed diameter and the number of observations in each pair of sets arranged according to the position angle of the lines employed. Table XII. gives the means, taken with reference to the number of observations, of the results of Table XI.

TABLE XI.

$p = 80°$		$p = 70°$		$p = 60°$		$p = 50°$		$p = 45°$		$p = 0°$	
59″.66	6	61″.61	17	62″.07	6	63″.73	6	62″.47	6	61″.50	6
62.36	6	59.17	4	61.03	6			61.28	6	61.13	6
		60.87	6	61.30	6			63.11	6	61.03	6
				62.76	6			62.76	6	61.03	17
				61.06	17			62.18	17	61.03	4
				60.67	4			62.51	4	59.91	6
										61.23	6
										61.64	8

TABLE XII.

Position Angle of Lines.	Observed Diameter.	No of Observations.	Diameter reduced to Mean Distance.
80°	61″.01	12	16″.14
70°	61.09	27	16.16
60°	61.41	45	16.24
50°	63.73	6	16.85
45°	62.33	45	16.49
0°	62.09	59	16.42
Mean	61.83	194	16.35

The mean value of the diameter from the 194 observations is,

$$D = 16''.35.$$

It is noteworthy that the results over the different lines, with the exception of that at 50°, which is based on only 6 observations, all give values less than that of 16″.61, adopted in the Berlin Jahrbuch, Nautical Almanac, and Connaissance des Temps, and that the lines of greatest position angle, which by this method would be expected to afford the most accurate results, give the smallest values of the series.

It appears to me that the method of Professor Rogers is not limited, in its application to the interior planets, to their transits over the sun's disk, or to times when the conditions permit the whole disk to be seen; but that it may, by an appropriate construction of the plate and arrangement of the observations, be employed at any time when they are near inferior conjunction, and that determinations both before and after conjunction will eliminate any errors peculiar to each elongation.

Let p be the position angle, counted from an assumed zero, of a line on the plate drawn from some point taken as a centre; the *true* position angle being $p + \Delta p$. Let D and δ be the diameter and declination of the planet; $\Delta\delta$ the difference of declination from the centre of the plate when it passes north, and $\Delta'\delta$ when it passes south of that centre; and t and t', the corresponding observed times when the planet's limb in its diurnal motion is tangent to the line. Then in the triangle formed by the planet's centre, the intersection of its path with the line, and the observed point of tangency, the distance between the first two points is,

$$\tfrac{1}{2}D \sec (p + \Delta p) = \tfrac{1}{2}D \sec p + \tfrac{1}{2}D \tan p \sec p \,\Delta p$$

where, Δp being small, the terms involving its squares are neglected.

If we imagine a line drawn from the centre of the plate at the angle p from the true position zero, we have, from the triangle formed by the actual and imaginary lines and the portion of the path of the planet's centre between them, the length of the intercepted path:

$$\Delta\delta \,\Delta p \sec p \sec (p + \Delta p) = \Delta\delta \,\Delta p \sec^2 p$$

If now we call T the time when the centre of the planet is on this imaginary line when the planet passes north, and T' the time for a corresponding position when the planet passes south of the centre, we have the general equations:

$$T = t + \frac{1}{15 \cos \delta}\left[\pm \tfrac{1}{2}D \sec p \pm \tfrac{1}{2}D \tan p \sec p \,\Delta p + \Delta\delta \sec^2 p \,\Delta p\right]$$

(1)

$$T' = t' + \frac{1}{15 \cos \delta}\left[\mp \tfrac{1}{2}D \sec p \mp \tfrac{1}{2}D \tan p \sec p \,\Delta p + \Delta'\delta \sec^2 p \,\Delta p\right]$$

p being reckoned as usual from 0° to 360° in the direction n. f. s. p.; the upper sign being used for the preceding, and the lower sign for the following limb.

Let t_1 and t_2 be the observed times when the planet, passing north of the centre, has either limb tangent to lines at the angles p and $-p$ from the assumed zero. Then the 1st equation of (1) gives:

$$T_1 = t_1 + \frac{1}{15 \cos \delta}\left[\pm \tfrac{1}{2} D \sec p \pm \tfrac{1}{2} D \tan p \sec p \, \Delta p + \Delta\delta \sec^2 p \, \Delta p\right]$$
(2)
$$T_2 = t_2 + \frac{1}{15 \cos \delta}\left[\pm \tfrac{1}{2} D \sec p \mp \tfrac{1}{2} D \tan p \sec p \, \Delta p + \Delta\delta \sec^2 p \, \Delta p\right]$$

If t_0 be the corresponding time for a third line drawn through the intersection of the other two and bisecting the angle between them, we shall have $p = 0$, and

$$T_0 = t_0 + \frac{1}{15 \cos \delta}\left[\pm \tfrac{1}{2} D + \Delta\delta \, \Delta p\right] \tag{3}$$

But we have $T_2 - T_0 = T_0 - T_1$. Hence, putting $\tau = t_2 + t_1 - 2\, t_0$, and noting that $(1 - \cos p) \sec p = \tan p \tan \tfrac{1}{2} p$,

$$\mp \tfrac{1}{2} D \tan p \tan \tfrac{1}{2} p - \Delta\delta \tan^2 p \, \Delta p = \tfrac{15}{2} \tau \cos \delta \tag{4}$$

In a similar way, when the planet passes south of centre, we get

$$\mp \tfrac{1}{2} D \tan p \tan \tfrac{1}{2} p - \Delta'\delta \tan^2 p \, \Delta p = \tfrac{15}{2} \tau' \cos \delta \tag{5}$$

The addition of (4) and (5) gives,

$$D = \mp \cot \tfrac{1}{2} p \left[\tfrac{15}{2} \cos \delta \, (\tau + \tau') \cot p + (\Delta\delta + \Delta'\delta) \tan p \, \Delta p\right]; \tag{6}$$

and their difference,

$$\Delta p = \tfrac{15}{2} \cos \delta \, (\tau' - \tau) \frac{\cot^2 p}{\Delta\delta - \Delta'\delta} \tag{7}$$

But we have also, putting $r = t_2 - t_1$, and $r' = t'_2 - t'_1$,

$$\begin{aligned} \Delta\delta &= \tfrac{15}{2} r \cos \delta \cot p - \tfrac{1}{2} D \, \Delta p \sec p \\ \Delta'\delta &= -\tfrac{15}{2} r' \cos \delta \cot p - \tfrac{1}{2} D \, \Delta p \sec p \end{aligned} \tag{8}$$

whence

$$\begin{aligned} \Delta\delta + \Delta'\delta &= \tfrac{15}{2} (r - r') \cos \delta \cot p \text{ (nearly)} \\ \Delta\delta - \Delta'\delta &= \tfrac{15}{2} (r + r') \cos \delta \cot p \end{aligned}$$

which substituted in (6) and (7) give finally,

$$D = \mp \tfrac{15}{2} \cos \delta \cot p \cot \tfrac{1}{2} p \left[\tau + \tau' - (\tau - \tau') \frac{r - r'}{r + r'}\right] \tag{9}$$

$$\Delta p = \frac{\tau' - \tau}{r + r'} \cot p \tag{10}$$

The last term in equation (9) disappears when the planet passes at equal distances north and south, and in general is inappreciable except when the error of position zero is large, or when the planet passes at very unequal distances north or south of the centre, which in practice need never occur.

Equation (9) consequently permits the determination of the diameter by observations on one limb only. As has been remarked, observations on the preceding limb before inferior conjunction, and on the following limb after it, may be expected to eliminate errors peculiar to the elongation.

It should be remarked that the quantity cot p cot $\frac{1}{2}$ p becomes unity for $p = 60°$, and for larger angles rapidly increases. In general the advantageous application of the method requires the use of lines at greater position angles than 60°.

In what precedes it has been assumed that the line corresponding to the time t_0 bisects the angle formed by the others, and also passes through their intersection. In ruling the plates for the observations of the diameter during the transit of Venus, these conditions may possibly not have been exactly fulfilled; since, as they did not affect the observations then contemplated, Professor Rogers did not especially attend to those points in the preparation of the plates. Any such errors may, however, be eliminated. Thus, if we put $a =$ the distance between the transit line on the plate from an imaginary line parallel to it passing through the point of intersection of the inclined lines, and Δi the inclination to a line bisecting the angle of the inclined lines, equation (3) becomes,

$$T_0 = t_0 + \frac{1}{15 \cos \delta} \left[\pm \tfrac{1}{2} D + \Delta\delta \ (\Delta p + \Delta i) \ + a \right]$$

and equation (9),

$$D = \mp \tfrac{15}{2} \cos \delta \cot p \cot \tfrac{1}{2} p \ \Big[\tau + \tau' - 2\, a - 2\, (\Delta\delta - \Delta'\delta)\, \Delta i$$
$$+ \frac{r - r'}{r + r'} (\tau' - \tau) \Big]$$

Since Δi changes sign by turning the plate 180° in position angle, and a changes sign by turning the other side of the plate toward the eye these sources of error may be determined or eliminated by arranging the observations with appropriate reversals.

To exemplify partially the use of this method, I avail myself of some observations of the following limb of Venus, on various days succeeding the recent transit, by Professor Rogers and myself. As

these were not arranged with a view to eliminate the possible errors involved in a and Δi, as it is the intention to do in the future, the results cannot be considered as having other than an illustrative value. The angle, 60°, was less than should properly be used for advantageous results, and the record gives no means of knowing in which series the plate was in the direct, and in which it was in the reversed position. The results are as follows.

TABLE XIII.

Date.	Number of Observations.	Diameter.	Observer.
1882			
Dec. 13–14	25	16″.39	W. A. R.
" 14–15	20	17.06	"
" 24–25	22	17.58	"
" "	21	18.30	S. C. C.
" 26–27	17	17.89	"

These results are of interest for comparison with those obtained on the same days by Professor Rogers from different observations. The series is not sufficient to determine the "irradiation constant."

In conclusion, the results may be summarized as follows: —

1. Observations of the four contacts by six observers.

2. The determination, by a photometer especially devised for the purpose, of the relative amounts of light received from the disk of Venus, from the sky near the sun's edge, and from the sun's centre. Denoting the last amount by 100.0, that received from Venus was 1.6, and that received from the sky 7.5. Contrary to expectation, Venus was thus shown to be distinctly darker than the adjacent sky, and this result was confirmed by direct observation.

3. The spectroscopic observations. These gave negative results, and showed that no marked absorption was caused by the atmosphere of Venus.

4. A careful determination of the diameter of Venus by a method not previously attempted, and the suggestion of an application of this method to planets when both limbs cannot be observed. The result obtained by Professor Rogers was 16″.10 from transits over inclined lines, and that obtained by Mr. Chandler was 16″.35, which would be reduced 0″.02 by using only the transits over inclined lines.